Least Squares Matrix Operations for Scaling Distance and Prism Constant

Note: a calculator capable of doing matrix calculations, or knowledge of formula usage in Excel, is required to perform the necessary equations

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Review Basic Matrix Operations

Matrix Transpose:

$$\mathbf{A} = \begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 4 \end{bmatrix} \qquad \mathbf{A}^T = \begin{bmatrix} 5 & 1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix}$$

Matrix Inverse:

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \qquad \mathbf{A}^{-1} = \begin{bmatrix} -0.1 & 0.3 \\ 0.4 & -0.2 \end{bmatrix}$$

$$\mathbf{A} * \mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I} = \text{(Identity Matrix)}$$

Matrix Equations: S = Scaling Distance and C = Prism Constant

$$\mathbf{F} = \begin{bmatrix} D_{A1} \\ . \\ . \\ . \\ . \\ . \\ D_{An} \end{bmatrix}_{n \times 1} \qquad \mathbf{L} = \begin{bmatrix} D_{H1} \\ . \\ . \\ . \\ . \\ . \\ D_{Hn} \end{bmatrix}_{n \times 1}$$

D_A = NGS Published Distances for Base Line **D_H = Base Line distances as observed**

$$\mathbf{A} = \begin{bmatrix} D_{A1} & -1 \\ . & -1 \\ . & -1 \\ . & -1 \\ . & -1 \\ . & -1 \\ D_{An} & -1 \end{bmatrix}_{n \times 2}$$

$$\mathbf{K}_{n \times 1} = \mathbf{L} - \mathbf{F}$$

$$\mathbf{N} = \mathbf{A}^T * \mathbf{A}$$

$$\mathbf{T} = \mathbf{A}^T * \mathbf{K}$$

$$\mathbf{X}_{2 \times 1} = \mathbf{N}^{-1} * \mathbf{T}$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{S\ (ppm)} \\ \mathbf{C\ (meters)} \end{bmatrix}_{2 \times 1}$$

> PPM: Scaling factor for EDMI

> Instrument Reflector Constant

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2006

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Calibration Base Line Completed Example:

			$D_{A\,n}$	$D_{H\,n}$
ID#	**From**	**To**	**Published**	**Observed**
1	150	300	149.9929	149.9899
2	300	150	149.9929	149.9905
3	150	600	449.9990	449.9916
4	600	150	449.9990	449.9849
5	150	1800	1649.9959	1649.9600
6	1800	150	1649.9959	1649.9728
7	300	600	300.0061	300.0003
8	600	300	300.0061	299.9984
9	300	1800	1500.0030	1499.9739
10	1800	300	1500.0030	1499.9906
11	600	1800	1199.9969	1199.9866
12	1800	600	1199.9969	1199.9858

F =

$$\begin{bmatrix} 149.9929 \\ 149.9929 \\ 449.9990 \\ 449.9990 \\ 1649.9959 \\ 1649.9959 \\ 300.0061 \\ 300.0061 \\ 1500.0030 \\ 1500.0030 \\ 1199.9969 \\ 1199.9969 \end{bmatrix}_{12 \times 1}$$

L =

$$\begin{bmatrix} 149.9899 \\ 149.9905 \\ 449.9916 \\ 449.9849 \\ 1649.9600 \\ 1649.9728 \\ 300.0003 \\ 299.9984 \\ 1499.9739 \\ 1499.9906 \\ 1199.9866 \\ 1199.9858 \end{bmatrix}_{12 \times 1}$$

A =

$$\begin{bmatrix} 149.9929 & -1 \\ 149.9929 & -1 \\ 449.9990 & -1 \\ 449.9990 & -1 \\ 1649.9959 & -1 \\ 1649.9959 & -1 \\ 300.0061 & -1 \\ 300.0061 & -1 \\ 1500.0030 & -1 \\ 1500.0030 & -1 \\ 1199.9969 & -1 \\ 1199.9969 & -1 \end{bmatrix}_{12 \times 2}$$

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K = L - F

-0.0030
-0.0024
-0.0074
-0.0141
-0.0359
-0.0231
-0.0058
-0.0077
-0.0291
-0.0124
-0.0103
-0.0111

12 x 1

N = A^T * A

13454977.32	-10499.9876
-10499.9876	12

2 x 2

T = A^T * K

-199.8148388
0.1623

2 x1

X = N^{-1} * T

-0.00001354482009	**S (ppm)**
0.0016732964	**C (meters)**

2 x 1

S (ppm) = 13.5 ppm > PPM: Scaling factor for EDMI
C (mm) = 1.7 mm > Instrument Reflector Constant

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